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ANALYSIS

Entropy, limits to growth, and the prospects for weak sustainability

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Abstract

In this paper, we analyze the consequences of mass and energy conservation and the second law of thermodynamics for economic activity. In contrast to former studies, we deduce our results formally from a general model of production and consumption. We show that in a static setting for economies containing irreversible processes, a non-zero resource input as well as non-zero emissions are necessary to sustain a positive level of consumption. We generalize this result to a dynamic setting and apply it to the growth discussion and the sustainability discourse. Thereby we show that limits to growth of production and consumption are likely to exist and that the concept of weak sustainability is either morally unattractive or physically infeasible.

Keywords: Mass and energy conservation; Entropy law; Growth; Sustainability

1. Introduction

Since the publication of "The Entropy Law and the Economic Process" (Georgescu-Roegen, 1971) the question whether physical laws like the entropy law or the conservation laws of mass and energy are relevant to economic analysis has given rise to disputes. Two major positions have developed.

The mainstream position has been formulated by R. Solow as "[. . .] everything is subject to the entropy law, but this is of no immediate practical importance for modeling what is, after all, a brief instant of time in a small corner of the universe" (Solow, 1997, p. 268). Thus mainstream economists acknowledge the existence of these laws, but they claim that these laws have no substantial consequences for economic analysis and can therefore be safely neglected.

This position has attracted much criticism, especially from ecological economists. Daly (1997), among others, argues that it is based on a misinterpretation of the entropy law and the conservation laws; in a form suitable for open systems, these laws do not only apply to the universe as a whole but to all systems that process mass or energy, including economic production and consumption activities. Furthermore, these laws have important consequences as they rule out the common model of a closed, nature-independent economy that can grow without limits.

The problem which of the above positions is the better description of reality is surely of importance. But

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an agreement on this problem seems to be out of sight. One reason is that although there has been a sometimes-heated debate (see Section 2), the arguments that derive notable consequences from the above physical laws are often imprecise and remain obscure to many economists. As R. Solow writes in response to the arguments raised by Daly (1997): "Precise statements, best cast (I think) in the form of transparent models, are better than grand, heart-felt pronouncements about these issues" (Solow, 1997, p. 268). Indeed, most contributions to this subject are either informal (at least from the mainstream perspective), like Ayres (1998), Kårberger and Månsson (2001), and Ruth (1999), or lack generality because they use specific models, as in Anderson (1987), Smulders (1995a), or Young (1991). Thus from the point of the mainstream economist, a rigorous proof that the entropy law and the conservation laws of mass and energy matter for economic analysis is still missing.

This paper aims to cast the arguments of Georgescu-Roegen, Daly, and other ecological economists into a "precise" and "transparent" form and to provide thereby a rigorous and general proof of the relevance of physical constraints for economic analysis. Most of our arguments have been put forward a number of times already. The contribution of our paper is that it provides a broadly applicable formal analysis of the consequences of the laws of thermodynamics for economic modeling. We use a simple but general analytical model to derive several consequences from the conservation laws of mass and energy and from the second law of thermodynamics. Our model is based only on these physical laws and the concept of a market clearing equilibrium. Thus our results apply to nearly all static and dynamic economic models.

We use these results to address two questions. First, we analyze under what conditions limits to growth exist and which assumptions are needed to avoid such a conclusion. Second, we relate our results to the concept of weak sustainability, which is probably the most prominent sustainability concept, and show that this concept is either ethically unattractive or physically infeasible.

The paper is organized as follows. In the next section, we shortly discuss the entropy law and the conservation laws of mass and energy and their treatment in the economic literature. We then propose a simple model that depicts these laws and derive several consequences for static and dynamic economic systems. In Section 3, we apply our results to the growth debate and in Section 4 to the sustainability discourse. Section 5 concludes.

2. Mass and energy conservation and the second law of thermodynamics

Since the work of Ayres and Kneese (1969) and of Georgescu-Roegen (1971), many studies have analyzed the consequences of thermodynamic laws for economic analysis. On the microeconomic level, Islam (1985) analyzes the consequences of the second law of thermodynamics, showing that it implies that the isoquants of a production process cannot comply with the often-used assumption of a Cobb–Douglas technology. Anderson (1987) formally includes mass and energy balances into a model of production. He derives several results for production theory and uses them to analyze the possibilities for growth and for environmental policy. He shows that in the context of his model, substitution possibilities between man-made capital and resources have to be limited. Baumgärtner (2004) extends this result by proving that the Inada conditions, which are often used in economic models of optimal growth, are inconsistent with the material balance principle and that this inconsistency is preserved during aggregation.

On a macroeconomic scale, Ayres and Kneese (1969) and Noll and Trijonis (1971) have introduced mass and energy balances into static input–output analysis. Perrings (1986) has extended this analysis to linear dynamic models, showing that these balance equations can lead to instabilities. However, mass and energy conservation alone, that is, without considering the second law of thermodynamics, do not challenge the fundamental concepts of economic analysis, see Krysiak and Krysiak (2003).

The possible consequences of the second law on a macroeconomic scale are subject to an ongoing dispute. Young (1991, 1994), and Kårberger and Månsson (2001) argue that although the second law may have consequences on a microeconomic scale, it is not relevant on a macroeconomic scale because the earth is an open system that imports "low entropy" by solar radiation. Furthermore, the capability of human beings to innovate provides a way to defer possible negative consequences of the second law to an unforeseeable future or even to avoid them completely. Smulders (1995a,b) show this formally in the context of an endogenous growth model that includes human capital. Finally, Young (1994) argues that most of the content of the second law is already included in the common assumptions on cost functions, which, for example, often place prohibitive costs on perfect recycling. Therefore including the second law into economic analysis will not lead to substantially differing results. A similar point is made in Bretschger and Smulders (2004), where it is shown that even with considering the entropy law, increasing resource prices can lead to *R* and *D* investments that are sufficient to sustain unlimited growth. Similar arguments can be found in Solow (1997) and Stiglitz (1997) or in Ayres (1998).

These points are disputed by Townsend (1992) and Daly (1992, 1997), who argue that they are based on an inaccurate implementation of the second law of thermodynamics. Especially, they argue that all feasible production processes are subject to the laws of thermodynamics, so that innovation will not provide a means to escape the constraints imposed by these laws.

So there is a considerable literature on this subject. But most of this literature is informal and the formal studies, with the exception of Baumgärtner (2004), employ assumptions on production possibilities that rule out many often used economic models. In the following, we use a model that encompasses most static and dynamic models commonly used in economics, to show formally which conclusions can be drawn from the laws of mass and energy conservation and the second law of thermodynamics.

Let us assume an economy that consists of n actors, where "actor" encompasses consumers and producers. In this economy m goods are produced and consumed. In order to apply physical constraints to our model, we distinguish goods in addition to their usual economic characteristics also by their physical state. For example, we take a bottle of water at ambient temperature to be a different good than a bottle of refrigerated water because both goods are at different physical states.[1]

[1] As long as m is finite (and given any non-infinitesimal lower bound to the possibility to distinguish different physical states, as it is, e.g., given by the limited accuracy of the SI units, m will be finite) the resulting extension of the commodity space does not influence our subsequent arguments.

We label the amount of the goods that are used by actor i by an m-dimensional vector y^i. The actors can also use q resources and generate v types of emissions. We denote the amount of resources used by actor i by a vector r^i and the emissions generated by this actor by a vector e^i. As with goods, we distinguish resources and emissions according to their physical state. We use the convention that inputs for actor i are characterized by positive values, whereas outputs are characterized by negative values. Consequently, we have $r^i \geq 0$ and $e^i \leq 0$ since resources are inputs and emissions are outputs. Furthermore, the vectors y, r, e shall be comprehensive, that is, any flow that is an input or output to some economic actor is included in one of these vectors. This assures that all connections between the economy and its natural surrounding are included in our model.

For the following analysis, we distinguish the goods according to two criteria. First, we distinguish them according to their physical characteristics: Goods can be physical, in the sense that they have a strictly positive mass or energy content. Or they can be non-physical, implying a zero mass and a zero energy content. Second, we distinguish them according to their production: Goods can be strictly irreversibly produced, in the sense that the production of an additional unit of the good always increases the entropy production of the production process. Or they can be eventually reversibly produced, in the sense that eventually a further expansion of production does not increase the entropy production of the process anymore. Note that the latter distinction differs from the characterization of reversible and irreversible production processes in thermodynamics in that it is a definition based on the *marginal* entropy production, whereas the thermodynamic definitions are based on the *total* entropy production. To point out this difference, we use these somewhat awkward terms throughout this paper.

We assume that all resources and emissions are physical in the sense that they have a strictly positive mass or energy content.

Let p_j^y be the mass of one unit of good y_j (which will be zero for non-physical goods), p_j^r the mass of one unit of resource r_j, and p_j^e the mass of one unit of emission e_j. We collect these values into a single vector $p^T := (p_1^y, \ldots, p_m^y, p_1^r, \ldots, p_q^r, p_1^e, \ldots, p_v^e)$. Similarly, let w be the vector of the energy contents of y^i, r^i, and e^i. All elements of p and w are non-negative and finite and, since resources and emissions are physical, we have either $p_j > 0$ or $w_j > 0$ for all emissions and resources j. Using our sign convention, we can write the equilibrium version of the conservation laws of mass and energy as

$$\left(y^{i^T}, r^{i^T}, e^{i^T}\right)p = 0, \tag{1}$$

$$\left(y^{i^T}, r^{i^T}, e^{i^T}\right)w = 0. \tag{2}$$

These equations state that in a flow-equilibrium, the mass of inputs used by actor i must equal the mass of his/her outputs and that the same is true for the energy content of the inputs and outputs. This has to hold for all actors, that is, for $i = 1\ldots, n$.

Apart from their mass and energy content, each good, resource, and emission is characterized by its entropy content. Although entropy is a state variable, such a characterization is possible by our distinction of goods according to their physical state and by assuming that the physical state of each good does not change between production and consumption. The latter assumption should be taken as implying only that all changes in the physical state of a good are modeled as taking place while the good is in the possession of some economic actor. Furthermore, we restrict our attention to the entropy concept that is used in engineering thermodynamics, see, for example, Bejan et al. (1996) or Moran and Shapiro (1995), and thereby neglect configurational entropy. Finally, we set the entropy content of non-physical goods to zero.[2]

As above, we use a vector, now labeled s, to denote the entropy content of the goods, resources, and emissions. All elements of s are non-negative and finite. However, in contrast to mass and energy, entropy is not conserved but can be produced in the processes of production or consumption. Therefore the entropy balance for actor i contains a non-negative production term $\sigma^i(y^i, r^i, e^i)$:

$$\left(y^{i^T}, r^{i^T}, e^{i^T}\right)s + \sigma^i\left(y^i, r^i, e^i\right) = 0. \tag{3}$$

Eq. (3) is an entropy balance equation for open systems in a flow-equilibrium, that is, in a steady state. Therefore it is applicable whenever the steady-state assumption applies (cf., e.g., Bejan et al. (1996, p. 58)). The possibility to offset an entropy generation by an entropy export, which is present in open systems, is included in Eq. (3) by the flows y^i, r^i, and e^i.

Eqs. (1) (2) and (3) are the physical basis of our model. In addition to these equations, we use only the assumption of a market clearing equilibrium, which in our notation implies $\sum_{i=1}^{n} y^i = 0$.

To derive results from our model, we aggregate the model from the above microeconomic context to a macroeconomic one. Let $y := \sum_{i=1}^{n} y^i$, $r := \sum_{i=1}^{n} r^i$ and $e := \sum_{i=1}^{n} e^i$, that is, y corresponds to the excess demand of goods. Similarly, the vector r denotes the total resource use and the vector e the total emissions of the economy.

Using this notation, the market clearing condition can be stated as $y = 0$. The aggregation of Eqs. (1) and (2) together with $y = 0$ yields

$$\left(0, r^T, e^T\right)p = 0, \tag{4}$$

$$\left(0, r^T, e^T\right)w = 0. \tag{5}$$

Now all elements of p and w are non-negative, for all emissions and resources j, we have either $p_j > 0$ or $w_j > 0$ and we have $r \geq 0$ and $e \leq 0$. Thus we can conclude from Eqs. (4) and (5) that a non-zero value for at least one emission is only possible if there is at least one non-zero resource input and vice versa.

Proposition 1. *In a market clearing equilibrium, emissions are only possible with a non-zero resource use, and a non-zero resource use always implies emissions.*

This result, which is derived, for example, in Ayres and Kneese (1969) or Anderson (1987) in a more specialized context, is very intuitive (and rather self-evident, it could be argued). Seen physically, an economy in a market clearing equilibrium is an open system in which resources are the inputs and emissions the outputs. Mass and energy conservation implies that there can be no outputs without inputs, and vice versa, and thus no emissions without resource use.

Proposition 1 does not rule out an economy that exists without resource use and emissions, for example, due to perfect recycling. To analyze the feasibility of such a nature-independent economy, we use the second law of thermodynamics.

[2] According to our definition, a non-physical good contains neither mass nor energy. Furthermore, we neglect configurational entropy. Thus setting the entropy content of non-physical goods to zero is appropriate in our setting.

Aggregating Eq. (3) and combining it with the market clearing condition $y=0$ yields

$$\left(0, r^T, e^T\right)s + S\left(y^1, \ldots, y^n, r^1, \ldots r^n, e^1, \ldots, e^n\right) = 0, \tag{6}$$

where we have set $S(y^1,\ldots, y^n, r^1,\ldots r^n, e^1,\ldots, e^n) := \sum_{i=1}^{n}\sigma^i(y^i, r^i, e^i)$.

The vector s consists only of non-negative elements and, by our sign convention, we have $r \geq 0$ and $e \leq 0$. From the second law of thermodynamics, we know that $S(y^1,\ldots, y^n, r^1,\ldots r^n, e^1,\ldots, e^n) \geq 0$ for all values of $y^1,\ldots, y^n, r^1,\ldots r^n, e^1,\ldots, e^n$. Consequently, Eq. (6) can only be met if either $S=0$ or if e is non-zero, which in turn implies that r is non-zero. The first case corresponds to a reversible economy. The definition of S and the second law of thermodynamics together imply that this case is only possible if all σ^i equal zero, that is, the economy is only reversible if the production and consumption processes of all actors are either reversible or remain unused. Thus if at least one irreversible production or consumption process is active, at least one emission and thus by Proposition 1, at least one resource input is necessary in an equilibrium.

Proposition 2. *If $S>0$, that is if at least one active production or consumption process in the economy generates entropy, the concept of a market clearing equilibrium implies that there has to be at least one non-zero emission and at least one non-zero resource input.*

Proposition 2 is almost self-evident from a physical point of view. For a given system in a flow-equilibrium, in which irreversible processes take place, the entropy that is generated within the system has to flow out by the means of a high-entropy output. By Proposition 1, such an output necessitates an input.

Proposition 2 yields an answer to the question of the feasibility of a nature-independent economy: Such an economy cannot be excluded, but all production and consumption processes in such an economy would have to be reversible; a single irreversible process necessitates emissions and resource use.

As a next step, we analyze how much emissions are generated and which amount of resources is needed to sustain a given level of economic activity. For this, we use the distinction of goods into eventually reversibly produced goods and strictly irreversibly produced goods. Denote the produced amount of good j by $z_j := \sum_{i=1, y_j^i>0}^{n} y_j^i$. By the definition of a strictly reversibly produced good, we have $S(z)/z_j \geq \alpha_j > 0$ for all $z_j > 0$, that is, the average entropy production is bounded away from zero.

This implies that more production of a strictly irreversibly produced good always implies a higher S. Therefore, we can conclude from Eq. (6) that more production of these goods necessitates more emissions in equilibrium and by Eqs. (4) and (5), thus, a higher resource use.

Proposition 3. *Under our assumptions, an increase in the production or consumption of strictly irreversibly produced goods increases the equilibrium resource use and the equilibrium emissions. An infinite amount of production and consumption of such goods is only possible with an infinite amount of resource inputs and an infinite amount of emissions.*

The second part of this proposition follows from our definition of a strictly irreversibly produced good, which implies $S \to \infty$ for $z_j \to \infty$.

Finally, we generalize our model to a dynamic setting, for discussing the existence of limits to growth and the feasibility of weak sustainability. For this, we have to account for the accumulation of stocks of factors of production and thus for investment. Since the market clearing condition is the only economic assumption used so far, including investment is the only change necessary to account for economic dynamics.

Let I_i denote the investment into a stock of good i. The condition for a market clearing equilibrium is thus $y^i = -I_i$. We assume $I \geq 0$, leading to $y \leq 0$. Thus we analyze only the accumulation of stocks of goods not the depletion of such stocks.[3] Since we do not use specific assumptions on production possibilities, this simple change is sufficient to cover nearly all forms of economic dynamics. Obviously it allows to model any kind of accumulation processes. Furthermore, our model can also account for changes to the produced and consumed goods by including all feasible (at present and in the future) goods in the vector y and by

[3] Given finite initial stocks, a process of depleting these stocks can only be transitory and is thus irrelevant for the long-run focus of the following sections.

allowing for a change in the composition of y over time. Thus our setting includes models in which the accumulation of human or physical capital leads to innovation, like the design of new goods and new production processes.

Due to $y = -I \leq 0$, it is not necessary any more that entropy generated by production or consumption has to be exported to the environment via emissions. Such entropy can simply be "stored" in the capital stocks and therefore it might be possible to increase the production of irreversibly produced goods without using more resources as long as the net investment into the capital goods is sufficiently large. In this way, the main argument of Propositions 2 and 3 is circumvented.

But a slightly more intricate argument suffices to show that such a process is infeasible. Consider a strictly irreversibly produced good j. Assume that an increase in the production of this good is possible without increasing the resource use. We will show that this assumption is not compatible with the physical constraints.

By the definition of a strictly irreversibly produced good, an increase in the production of good j increases the total entropy production S by a non-infinitesimal amount. With capital accumulation the aggregated entropy balance reads

$$\left(-I^T, r^T, e^T\right)s + S\left(y^1, \ldots, y^n, r^1, \ldots, r^n, e^1, \ldots, e^n\right) = 0. \tag{7}$$

By assumption, r remains constant. Therefore an increase in S necessitates an increase in some element of I (note that we have $I \geq 0$ and $e \leq 0$). This increase has to be in an element of I with non-zero entropy content. Now the aggregated mass and energy balances with capital accumulation are

$$\left(-I^T, r^T, e^T\right)p = 0, \tag{8}$$

$$\left(-I^T, r^T, e^T\right)w = 0. \tag{9}$$

So increasing I while keeping r constant is only possible if the elements of p and of w that correspond to the increased elements of I are zero. But since such goods have zero entropy content, this contradicts the assumption needed to meet the entropy balance. So Eqs. (7) (8) and (9) cannot hold simultaneously for an increase in the production of an irreversibly produced good without an increase in r.[4]

Proposition 4. *Under our assumptions, an increase in the production or consumption of strictly irreversibly produced goods increases the resource use, even if capital accumulation is possible. An infinite amount of production and consumption of such goods is only possible with an infinite amount of resource inputs.*

The intuition behind this proposition is simple. It is possible to use accumulation processes to "store" entropy that is generated in a production process. But for increasing production this implies that the net investment has to be increased (the stock has to be made larger to store the additional entropy). But due to the material and energy balances, a larger net investment implies more resource use.

With Propositions 1–4, we are now in a position to inquire about the implications of physical constraints for growth and sustainability.

3. Is unlimited growth physically feasible?

We now apply our results to analyze whether the finiteness of the earth and of the solar radiation reaching the earth imply limits to economic growth or whether unlimited growth is possible. In order to gain broadly applicable results, we use a growth model that includes most of the commonly used economic growth models, like the Solow model or the endogenous growth models of the Romer type.

We start with the environment. Here we simply assume that the available flow of resources into the economy is bounded, that is, the environment has a limited regeneration capacity. This assumption depicts the finiteness of the earth and the limited amount

[4] It could be argued that this conclusion is only a consequence of setting the entropy content of non-physical goods to zero, that is, of neglecting configurational entropy. But to use this argument for rejecting the conclusions of the following sections, it would be necessary to take the view that it is possible to store the eventually infinite amount of entropy generated by the unlimited growth of the production of strictly irreversibly produced goods in the form of configurational entropy, that is, in a stock of non-physical goods. This seems to be a substantial exaggeration of the importance of configurational entropy.

of solar radiation reaching the earth. It is hardly controversial.

Concerning the economic part of the model, we assume that all goods can be accumulated and the resulting stocks used as factors in production. Treating human capital as a non-physical good thus allows for human capital accumulation in this model.

The goods are produced according to production functions $f_i(Z, r, e, l)$ that depend on the vector of stocks of goods Z, the vector of resource use r, the vector of emissions e, and a labor input l. We only assume that these functions are continuous with respect to all of their variables and that the labor input can vary, due to demographic changes, but that it is finite at all times.

We assume that the stocks of capital goods Z resemble machinery or human capital and are thus not part of the product. Consequently they do not enter the mass, energy, and entropy balances of the production process. The only change is that a part of the final product is not consumed but invested.

Let $\phi_i(t)$ be the part of total production of good i that is invested at time t. By definition, $\phi_i(t) \in [0, 1]$ for all t and for all $i \in \{1,\ldots, m\}$. We make no behavioral assumptions concerning investment; $\phi_i(t)$ can take on arbitrary values (in the specified range) over time. Thus the accumulation process can be written as

$$\dot{Z}_i = \phi_i(t) f_i(Z, r, e, l), \ \forall i \in \{1, \ldots, m\}. \qquad (10)$$

Eq. (10) specifies a very general growth model that includes most of the commonly used models. Especially, it encompasses the models used in Smulders (1995a,b), who, among other points, analyzed the possibilities for unlimited growth in the presence of physical constraints.

For analyzing the implications of Proposition 4 for economic growth, it is helpful to identify those goods whose stocks have to be increased to allow for an increase in the production of a given good. The following definition serves this need.

Definition 1. Let A be a subset of $\{1,\ldots, m\}$ with the property that for fixed, finite values of r, e, l, and of all Z_k with $k \notin A$, we can have $f_i(Z, r, e, l) \to \infty$ for all $i \in A$ by letting some (or all) $Z_j \to \infty$ with $j \in A$. Then the goods with index in A are *decoupled* from the rest of the economy.

In words, the goods collected in set A can be produced in arbitrary quantities with only increasing the stocks of capital goods that are in the same set A. The part of the economy that produces the goods in A is decoupled from the rest of the economy; it can grow even if the rest stagnates.

Let us now consider the prospects of unlimited growth in our model. With unlimited growth we denote a solution of Eq. (10) for finite starting values of all Z_i in which at least one f_i goes to infinity over an infinite time horizon.

Due to the continuity of the production functions, $f_i \to \infty$ requires that some input becomes infinite. By assumption, the resource input into the economy is bounded and, by Eqs. (8) and (9), this implies a limit to the amount of emissions. The labor input is also finite, so that an infinite production can only be possible with an infinite physical or human capital input.

Now, Proposition 4 states that for the case of irreversibly produced goods, an infinite production requires an infinite resource use, regardless of the size of the stocks Z. By our above analysis, this is not possible. Thus only eventually reversibly produced goods can be produced in infinite quantities. Furthermore, we have to assure that, for such an infinite production, no strictly irreversibly produced inputs are needed in infinite quantities. So for unlimited growth, there has to exist a subpart of the economy that includes only eventually reversibly produced goods and that is decoupled from the rest of the economy.

Proposition 5. *Under our assumptions, unlimited growth is only possible if there exists a subset A of goods that includes only eventually reversibly produced goods and that is decoupled from the rest of the economy, in the sense of Definition 1. In such a case, unlimited growth is restricted to the goods included in A.*

Proposition 5 shows that if physical constraints are accounted for, the prospects of unlimited growth are severely diminished. For unlimited growth, there has to exist a part of the economy that uses only production technologies with a vanishing average entropy production in the limit and this part has to be decoupled from the rest of the economy in the sense that it can grow while the rest stagnates.

This result is driven only by the physical constraints and our assumptions on a limited regeneration capacity of the environment; no behavioral assumptions and no additional assumptions on the production technology are needed. So the discrepancy between Proposition 5 and the common results of economic growth theory are due to the physical constraints.

Note, however, that some care is necessary to interpret Proposition 5. We have distinguished goods according to their production processes and allowed for the inclusion of future production processes in this distinction. Thus in the commonly used modeling context, which does not use this distinction of goods, our proposition must be read as referring to goods that are at all times produced by strictly irreversible processes. But since at least for physical goods the assumption of an eventually reversible production process is very restrictive, this point will not seriously limit the applicability of our results.

Given that our model includes those of Smulders (1995a,b), it is interesting to compare the results. There are two reasons why the models of these studies allow for unlimited growth. First, they include human capital as an eventually reversibly produced good. Second, they assume constant returns to scale w.r.t. man-made inputs. By Proposition 4, the latter assumption can only apply to eventually reversibly produced goods. So these studies do not include strictly irreversibly produced goods at all. Therefore growth cannot be limited by physical constraints in these studies.

Finally, Proposition 5 shows that it is the marginal rather than the total entropy production that is relevant for the question of limits to growth. Also it is necessary to consider both the entropy law and the conservation laws; studies that use only either of these cannot derive limits to growth.

4. The concept of weak sustainability

The growth debate of the 1970s has been succeeded by the sustainability discussion. It is therefore interesting to inquire whether the sustainability discussion is based on physically more plausible assumptions than growth theory.

There are many concepts of sustainability, but only a few are widely discussed. Especially prominent are the concepts of weak and strong sustainability. Many arguments have been put forward in favor of one or the other concept. But with the exception of Gutès (1996), who argues that weak sustainability is based on similar assumptions as the Solow–Hartwick model of growth, these arguments have not taken into account that these concepts may differ with respect to their physical feasibility.

Weak sustainability holds that each generation has the moral obligation to keep the total capital stock at least constant, where the total capital stock is comprised of the stocks of natural and produced capital. We can formalize this by defining a total capital stock C that is an aggregate of resource stocks x_i (comprised of the stocks of exhaustible and renewable resources) and the stocks of capital goods Z_i. Furthermore, such an aggregate is commonly taken to be a linear aggregate, that is, we have $C=\sum_{i=1}^{q+v}\beta_i x_i+\sum_{j=1}^{n}\gamma_j Z_j$, where β_i and γ_j are constant weights attached to the different stocks. This form of aggregation implies an infinite elasticity of substitution between the different stocks, that is, it is always possible to exchange one unit of resource stock i for β_i/γ_j units of capital stock Z_j leaving the aggregate C unchanged.

In our setting, the important questions are under which conditions this property does not devaluate the aggregate C as a measure for sustainability and whether these conditions are consistent with physical constraints.

Whether the aggregate C is a reasonable measure for sustainability amounts to the question if keeping this aggregate at least constant assures that future generations are not deprived of the means to meet their needs. Assuming that the needs of future generations will include some level of production of a fixed set of goods (e.g., food, shelter, and basic health services), sustainability would imply that the possibility to produce these goods has to be assured by keeping the aggregate C constant. Due to the linear form of this aggregate, the total depletion of the resource stocks (possibly reducing their regeneration capacity to zero) is allowed, if the stocks of man-made goods are increased to some finite value. So future generations are only guaranteed the possibility to meet their needs if this is possible without using resource inputs.

Can this condition hold in the presence of physical constraints? The answer is given by Eqs. (8) and (9) and by Proposition 2. The equations imply that without resource use, there cannot be capital accumulation.[5] Thus Proposition 2 can be applied, stating that it is only assured that future generations can meet their needs, if these needs include *only* goods that are produced by reversible processes.

Such a reversibility condition cannot be rejected on theoretical grounds. But it seems at least questionable whether a concept that shall serve to protect future generations should be based on a rather optimistic assumption that does not even hold at the present time. So the concept of weak sustainability, as it is commonly used, is either based on a physically inconsistent model or ethically unattractive, in the sense that it guarantees future generations the possibility to meet their needs only under rather optimistic assumptions on future technologies or preferences.

However, it should be clear that this is not a failure of the concept of keeping some aggregate stock constant but rather a consequence of an unfortunate choice of an aggregation rule. A better choice might be to choose a non-linear aggregate that takes into account the physical constraints by setting a lower bound to the resource stock. The idea of such isoquants can be found in Islam (1985) and in Ruth (1999). Such a revised concept of weak sustainability would closely resemble the usual definition, but it would incorporate sufficient physical information to assure that continued production and consumption is physically feasible along a sustainable path. Furthermore, since strong sustainability can be seen as a special form of a non-linear aggregation rule (a Leontief aggregate), such a concept of sustainability would provide a means to interpolate between the concepts of weak and strong sustainability.

5. Conclusions

In this paper, we have formally analyzed the consequences of physical conservation laws and the second law of thermodynamics for production and consumption. We have shown that in a static setting, these physical laws imply that economic activity is likely to depend critically on natural resources and on the ability of the environment to absorb generated emissions. Without either of these, no production or consumption is possible, except for goods that are produced and consumed by completely reversible processes. In a dynamic setting, the physical constraints imply that, even with the possibility to accumulate human or physical capital, more production of a good with non-vanishing marginal entropy production always necessitates more resource use.

We have related these findings to the debate about limits to growth by showing that such limits are likely to exist, if growth is taken to mean increasing production and consumption of physical goods. This result is derived from a growth model that incorporates most of the currently used models. Unlimited growth can be possible for goods with a vanishing marginal entropy production but only if these goods comprise a subpart of the economy that can grow autonomously. Nevertheless, our results indicate only that limits to growth for the production of most physical goods are likely to exist, they do not quantify these limits and thus do not imply that such limits will be met in the foreseeable future.

Our results also have implications for the sustainability discourse. We have shown that the concept of weak sustainability is either physically infeasible or ethically unattractive. If this concept shall be morally attractive it has to be amended by using a physically plausible aggregation rule.

As already discussed in the Introduction, many of the above results are "common sense" in ecological economics. The contribution of our analysis is that it provides a formal proof of these results that is based on a fairly general model. A further advantage of this approach is that it provides a clear focus on the consequences of the physical constraints. Apart from a few commonly used assumptions, our results depend only on the laws of thermodynamics. Thus in contrast to studies that introduce physical constraints into detailed economic models, like Anderson (1987), Smulders (1995a), or Young (1991), our conclusions are pure consequences of these laws; they do not depend on restrictive additional assumptions.

[5] The above aggregation rule implies that finite stocks of capital goods suffice to compensate for the depletion of the resource stocks. So it is not possible that all future generations live by depleting the capital stocks, i.e., $I<0$ is excluded.

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